Mathematics Formulae & some basic concepts

Class- X

Real Number

Euclid's Division Algorithm(lemma) : Given positive integers `a' and `b', there exists unique integers q and r such that $a = b.q + r$, where $0 \le r < b$ (where a = dividend, b = divisor, q = quotient, and r = remainder.

Polynomials

In step1 : Factorize the given polynomials,

a) Either by splitting the terms, (OR)

b) Using these identities :

(i) $(a + b)^2 = a^2 + 2ab + b^2$ **(ii)** $(a - b)^2 = a^2 - 2ab + b^2$ **(iii)** $a^2 - b^2 = (a + b)(a - b)$

(iv) $a^4 - b^4 = (a^2)^2 - (b^2)^2 = (a^2 + b^2)(a^2 - b^2) = (a^2 + b^2)(a - b)(a + b)$

(v) $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$ **(vi)** $a^3 + b^3 = (a + b)(a^2 + ab + b^2)$

(vii) $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$ **(viii)** $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

(ix) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

(x) $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$

Trial & Error method.

In step2 : Take the product of 'Common terms' as their HCF.

In step3 : Take the product of All the terms , Omit, the HCF value which gives you the value of LCM.

Product of LCM × HCF = Product of the two polynomials.

Note: If cubical expression is given, it may be factorized by using 'Trial & Error" method.

Remainder theorem

If $(x - 2)$ is a **factor** of the given expression, then take $x-2 = 0$,therefore $x = 2$, then substitute this value in $p(x) = 5x^2 + 3x - 6$ as

$p(2) : 5(2)^2 + 3(2) - 6 = 0$ (Here taking =0 is very important. If not taken answer can't be found.)

If (x-2) leaves a remainder of 4

$p(2) : 5(2)^2 + 3(2) - 6 = 4$ (Here taking =4 is very important. If not taken answer can't be found.)

Linear Equation in two variables

If pair of linear equation is : $a_1 + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$

Then nature of roots/zeroes/solutions :

(i) If $a_1/a_2 \ne b_1/b_2$ → system has unique solution, is consistent OR graph is two intersecting lines.

(ii) If $a_1/a_2 = b_1/b_2 \ne c_1/c_2$ → system has no solution, is inconsistent OR graph are parallel lines.

(iii)If $a_1/a_2 = b_1/b_2 = c_1/c_2$ → system has infinite solution, is consistent OR graph are coincident lines.

Quadratic Equations

Note: To find the value of 'x' you may adopt either 'splitting the middle term' or 'formula method',

$x = \frac{-b \pm \sqrt{D}}{2a}$ (where $D = b^2 - 4ac$) Hence $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

- Sum of the roots = $-b/a$ & Product of roots = c/a
- If roots of an equation are given, then :

Quadratic Equation : $x^2 - (\text{sum of the roots}).x + (\text{product of the roots}) = 0$

If Discriminant > 0, then the roots are Real & unequal or unique, lines are intersecting.

Discriminant = 0, then the roots are Real & equal, lines are coincident.

Discriminant < 0, then the roots are Imaginary (not real), parallel lines

If speed of boat = x km/hr and that of stream = y km/hr then speed in **upstream = (x–y)** km/hr and speed in **downstream = (x+y)** km/hr.

Ratio & Proportion

- Duplicate ratio of $a : b$ is $a^2 : b^2$ (Incase of Sub-duplicate ratio you have to take 'Square root')
- Triplicate ratio of $a : b$ is $a^3 : b^3$ (Incase of Sub-triplicate ratio you have to take 'Cube root')
- Proportion $\underset{1^{st}}{a} : \underset{2^{nd}}{b} = \underset{3^{rd}}{c} : \underset{4^{th}}{d}$ proportionals, Continued Proportion $\underset{1^{st}}{a} : \underset{2^{nd}}{b} = \underset{2^{nd}}{b} : \underset{3^{rd}}{c}$ proportionals, (Middle value is repeated)
- Product of 'Means'(Middle values) = Product of 'Extremes'(Either end values)
- If $\frac{a}{b} = \frac{c}{d}$ is given, then Componendo & Dividendo is $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

Note : " Where to take "K" method ?" You may adopt it in the following situations.

If a/b = c/d = e/f are given, then you may assume as a/b = c/d = e/f = k

Therefore a = b.k, c = d.k, e = f.k, then substitute the values of 'a' 'b' and 'c' in the given problem.

Incase of continued proportion : a/b = b/c = k hence, a = bk, b = ck therefore putting the value of b we can get $a = ck^2$ & b = ck.(putting these values equation can be solved)

Similarity

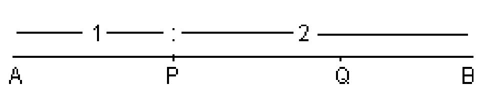
- If two triangles are similar then, ratio of their sides are equal.
  i.e if $\Delta ABC \sim \Delta PQR$ then $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$.
- If $\Delta ABC \sim \Delta PQR$ then $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \frac{\text{Side}^2}{\text{Side}^2} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$

Distance & Section Formulae

- Distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. (The same formula is to be used to find the length of line segment, sides of a triangle, square, rectangle, parallelogram etc.,)
- To prove co-linearity of the given three points A,B, and C, You have to find length of AB, BC, AC then use the condition AB + BC = AC. **OR** use this condition to solve the question easily :

Area of triangle formed by these points : $\frac{1}{2}.[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

- Section formula: point (x, y) = $\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$
- Mid point = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
- Centroid of a triangle = $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$



- If line is trisected then take m:n ratio as 1:2 and find co-ordinates of point p(x,y).

Equation of a line

- If two points are given, then Slope (m) = $\frac{y_2 - y_1}{x_2 - x_1}$
- If a point, and slope are given, then Slope (m) = $\frac{y - y_1}{x - x_1}$
- If two lines are 'Parallel' to each other then their slopes are equal i.e $m_1 = m_2$
- If two lines are 'Perpendicular' to each other then product of their slopes is – 1. i.e $m_1 \times m_2 = -1$
- Depending upon the question You may have to use equation od straight line as

a) y = mx + c, where 'c' is the y-intercept. OR b) $(y - y_1) = m.(x - x_1)$

Circles, & Tangents.

- Equal chords of a circle are equidistant from the center.(Chord Property)
- The perpendicular drawn from the centre of a circle, bisects the chord of the circle. (Chord Property)
- The angle subtended at the centre by an arc = Double the angle at any part of the circumference of the circle.(Angle Property)
- Angles subtended by the same arc in the same segment are equal. (Angle Property)
- To a circle, If a tangent is drawn and a chord is drawn from the point of contact, then angle made between the chord and the tangent = Angle made in the alternate segment.(Tangent Property)
- The sum of opposite angles of a cyclic quadrilateral is always 180°.

Circumference & Area of a Circle

- Area of a Circle = πr^2.
- Perimeter of a Circle = $2 \pi r$
- Area of sector = $\theta/360°(\pi r^2)$
- Length of an arc = $\theta/360°(2\pi r)$
- Area of ring = $\pi.(R^2 – r^2)$
- Distance moved by a wheel in one revolution = Circumference of the wheel.
- Number of revolutions = $\dfrac{\text{Total distance moved}}{\text{Circumference of the wheel.}}$

Area of an equilateral triangle = $\sqrt{3}/4.(\text{side})^2$.

Note: While solving 'Mensuration' problems, take care of the following.

1. If diameter of a circle is given, then find the radius first
   (Have you made mistake earlier by taking 'd' as 'radius' and solved the problem ?)

2. Check the units of the entire data. If the units are different, then convert them to the same units.
   For Example: Diameter = 14 cm, and Height = 3 m
   Therefore Diameter = 14 cm, and Height = 300 cm (Have you ever committed such mistake ?)

Solids

1. Cylinder: Volume of a cylinder = $\pi r^2 h$
Curved surface area = $2 \pi r h$
Total surface area = $2 \pi r h + 2\pi r^2$ = $2 \pi r (h + r)$
Volume of hollow cylinder = $\pi R^2 h – \pi r^2 h$ = $\pi (R^2 – r^2) h$
TSA of hollow cylinder = Outer CSA + Inner CSA + 2 · Area of ring.
$2 \pi R h + 2 \pi r h + 2 \cdot [\pi R^2 – \pi r^2]$
(Of course, If you want, you may take 2π 'common')

2. Cone: Volume of a Cone = $\frac{1}{3} \pi r^2 h$.
CSA of a Cone = $\pi r \ell$ (Here 'ℓ' refers to 'Slant height') [where $\ell = \sqrt{(h^2 + r^2)}$]
TSA of a Cone = $\pi r \ell + \pi r^2$ = $\pi r (\ell + r)$

3. Sphere: Surface area of a Sphere = $4\pi r^2$. (In case of Sphere, CSA = TSA i.e they are same)
Volume of hemi sphere = $\frac{2}{3} \pi r^3$ [Take half the volume of a sphere]
CSA of hemisphere = $2 \pi r^2$ [Take half the SA of a sphere]
TSA of hemisphere = $2 \pi r^2 + \pi r^2 = 3 \pi r^2$
Volume of a Sphere = $\frac{4}{3} \pi r^3$

Volume of spherical shell = Outer volume – Inner volume = $4/3.\pi.(R^3 - r^3)$

While solving the problems based on combination of solids it would be better if you take common.

- T.S.A. of combined solid = C.S.A of solid 1 + C.S.A of solid 2 + C.S.A of solid 3
- If a solid is melted and, recast into number of other small solids, then
  Volume of the larger solid = No of small solids x Volume of the smaller solid
  For Ex: A cylinder is melted and cast into smaller spheres. Find the number of spheres
  Volume of Cylinder = No of sphere x Volume of sphere.
- If an 'Ice cream cone with hemispherical top' is given then you have to take
  a) Total Volume = Volume of Cone + Volume of Hemisphere
  b) Surface area = CSA of Cone + CSA of hemisphere (usually Surface area will not be asked)

Trigonometric Identities

- Wherever 'Square' appears think of using the identities

(i) $\sin^2\theta + \cos^2\theta = 1$ (ii) $\sec^2\theta - \tan^2\theta = 1$ (iii) $\text{Coseec}^2\theta - \cot^2\theta = 1$

- Try to convert all the values of the given problem in terms of Sin θ and Cos θ
- Cosec θ may be written as 1/Sin θ — Sin θ. Cosec θ = 1
- Sec θ may be written as 1/Cos θ — Cos θ. Sec θ = 1
- Cot θ may be written as 1/Tan θ — Tan θ. Cot θ = 1
- Tan θ may be written as Sin θ / Cos θ — Cot θ = Cos θ / Sin θ
- Wherever fractional parts appears then think taking their 'LCM'
- Think of using $(a+b)^2$, $(a-b)^2$, $(a+b)^3$, $(a-b)^3$ formulae etc.,
- Rationalize the denominator [If a + b, (or) a – b format is given in the denominator]
- You may separate the denominator For Ex : <u>Sin θ + Cos θ</u> as <u>Sin θ</u> + <u>Cos θ</u> = 1 + Cot θ

- <u>If you are not able to solve the LHS part completely, Do the problem to such an extent you can solve, then start working with RHS, and finally you will end up at a step where LHS = RHS</u>
- $\sin(90-\theta) = \cos\theta$: $\cos(90-\theta) = \sin\theta$.
- $\sec(90-\theta) = \text{cosec}\,\theta$: $\text{cosec}(90-\theta) = \sec\theta$
- $\tan(90-\theta) = \cot\theta$: $\cot(90-\theta) = \tan\theta$

Values of Trigonometric Identities :

	0°	30°	45°	60°	90°
Sin θ	0	1/2	1/√2	√3/2	1
Cos θ	1	√3/2	1/√2	1/2	0
Tan θ	0	1/√3	1	√3	∞

Graphical Representation

- Don't forget to write the scale on x-axis, and on y-axis.
- To find the 'Lower quartile' take N/4 [Here N is $\sum f$] then take the corresponding point on X-axis
- To find the 'Upper quartile' take 3N/4, then take the corresponding point on X-axis
- To find the 'Median' take N/2, then take the corresponding point on X-axis

Measures of Central Tendency

For un-grouped data

- Arithmetic Mean = $\dfrac{\text{Sum of observations}}{\text{No of observations}}$
- Mode = The most frequently occurred value of the raw data.

- To find the Median first of all arrange the data in 'Ascending' or 'Descending' order, then
  Median = (N+1)/2 term value of the given data, in case of the data is having odd no of observations.
  Median = [(N/2) + (N+1)/2)] / 2 term value of the given data, in case of the data is having even number of observations.

  For grouped data

  Arithmetic Mean $= \dfrac{\sum fx}{\sum f}$ (Direct method)

  Arithmetic Mean $= a + \dfrac{\sum fd}{\sum f}$ (short cut method)

  Arithmetic Mean $= a + \dfrac{\sum fu}{\sum f} \times C$ (where C is class interval) (step-deviation method)

Probability

Probability of an event : P(event) $= \dfrac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$

If probability of happening an event is x then probability of not happening that event is (1-x).
For e.g. If probability of winning a game is 0.4 then probability of loosing it is (1-0.4) = 0.6
If probability of finding a defective bulb is 3/7 then probability of finding a non-defective bulb is (1-3/7) = 4/7.
In a deck of playing cards, there are four types of cards :

♠ (Spades in Black colour) having A, 2,3,4,5,6,7,8,9,10,J,K, and Q total 13 cards
♣ (Clubs in Black colour) having A, 2,3,4,5,6,7,8,9,10,J,K, and Q total 13 cards
♥ (Hearts in Red colour) having A, 2,3,4,5,6,7,8,9,10,J,K, and Q total 13 cards
♦ (Diamond in Red colour) having A, 2,3,4,5,6,7,8,9,10,J,K, and Q total 13 cards
52 cards

- Jack, King and Queen are known as 'Face Cards' , As these cards are having some pictures on it. Always remember **Ace is not a face card** as it doesn't carry any face on it.
- If one coin is tossed the total number of outcomes are 2 either a Head or a Tail.
- If two coins are tossed the total number of outcomes are 2 ×2 = 4
- If three coins are tossed the total number of outcomes are 2 ×2×2 = 8
- Similarly for Dice, In a single roll total number of outcomes are 6
- If two Dices are rolled, total number of outcomes are 6×6 = 36